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The isotherms migration method in the theory and practice of heat and mass transfer investigation—I. Kinematics of temperature fields

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Abstract—The process of unsteady heat conduction in bodies with and without phase transition of the substance is considered in terms of the migration of isothermal surfaces. The equations are derived to determine the velocity of their motion in a half-space and in limited bodies at the thermophysical characteristics which are dependent on temperature. In this case a new common feature of thermal kinetics regularization that is independent of the velocity of the isotherms motion of time is found. The independence of this quantity in central parts of limited bodies of a thermal situation on their outer surfaces is also revealed. The effect of non-linear boundary conditions on the kinetics of the temperature fields by the radiation law is considered. The regularities and the specific features of temperature fields formation found form the basis for the identification of the heat transfer coefficient and the dependence of thermal diffusivity on temperature.

1. INTRODUCTION

ON SOLVING linear and non-linear problems of the heat and mass transfer theory, on determing the thermophysical characteristics of the body material under conditions of its heat exchange with the surroundings, a non-traditional treatment of the studied phenomenon turns out to be useful. As will be shown, the idea of considering the process of non-stationary heat conduction in terms of the migration of isothermal (isopotential) surfaces rather than in terms of spatial–temporal temperature variation proves to be fruitful. In this case the problems of non-stationary heat conduction with and without phase transition are combined, it appears possible to find a number of new regularities of the processes studied and new ways are opened for solving non-linear problems.

Such an approach was shown when studying mass transfer in capillary-porous bodies [1, 2]. In the then published paper by Dix and Cizek [3] the one-dimensional equation of heat conduction in the migration of isotherms was obtained and some problems of its numerical solution were considered. A great series of works [4–20] by this author made it possible to reveal, in terms of the migration of isotherms, a number of new specific features of non-stationary heat conduction that were applied to the determination of the thermophysical properties of the body material, of the parameters of the boundary conditions, to numerical and analytical solutions of non-linear problems with and without phase transition of the substance. The problem under consideration attracts the ever growing interest of researchers [21, 22].

2. REGULARITIES OF THE MIGRATION OF ISOTHERMAL SURFACES IN BOUNDED BODIES WITH CONSTANT THERMOPHYSICAL PROPERTIES

It is evident that the migration of isothermal surfaces in a solid body should be characterized by a certain velocity v, the relation for which may be obtained from the following considerations.

As is known a non-stationary one-dimensional temperature field in bodies of the simplest form—a plate ($m = 1$), a cylinder ($m = 2$), a sphere ($m = 3$)—is found from the solution of the differential equation

$$C\frac{\partial T}{\partial \tau} = \frac{\lambda}{x^{m-1}}\frac{\lambda}{\partial x}\left[x^{m-1}\frac{\partial T}{\partial x}\right], \quad \tau > 0, \quad 0 < x < l_0 \tag{1}$$

with the corresponding boundary value conditions.

Satisfy equation (1), representing a symmetric development of a temperature field in the form of a series built by even powers of the coordinate x

$$T(x, \tau) = \sum_{n=0}^{\infty} A_{2n}(\tau)x^{2n}. \tag{2}$$

Then, bearing in mind that for an isothermal surface, $T = \text{const.}$, its position x in a body is variable with time τ, calculate the differential of both sides of equation (2)

$$0 = \sum_{n=0}^{\infty} A'_{2n}(\tau)x^{2n}\,d\tau + \sum_{n=1}^{\infty} 2nA_{2n}(\tau)x^{2n-1}\,dx,$$

NOMENCLATURE

a $= \lambda/C$, thermal diffusivity [$m^2 s^{-1}$]
Bi $= \alpha l_0/\lambda$, Biot number
C, $C(T)$ volumetric heat capacity [$J m^{-3} K^{-1}$]
c_0 $= 5.67$ W m^{-2} K^{-4}, emissivity of an absolutely black body
Fo $= a\tau/l_0^2$, Fourier number
l_0 characteristic dimension of a body (plate half-thickness, radius of cylinder, sphere) [m]
m coefficient of body shape equal to unity, two or three for plate, cylinder and sphere, respectively
Sk $= \varepsilon c_0 l_0 T_r^3 10^{-8}/\lambda$, Stark number
$T(x, \tau)$ current temperature [K]
T_f temperature of surrounding medium [K]
T_0 initial temperature [K]
T_r radiator temperature [K]
v, $\bar{v} = v\,l_0/a$ dimensional ($m\,s^{-1}$) and dimensionless velocities of migration isotherms
x coordinate [m].

Greek symbols
α coefficient of heat transfer [$W m^{-2} K^{-1}$]
ε emissivity of body surface
θ $= (T(x,\tau)-T_0)/(T_f-T_0)$, $\vartheta = T(x,\tau)/T_r$, dimensionless temperatures
λ, $\lambda(T)$ thermal conductivity [$W m^{-1} K^{-1}$]
ξ $= x/l_0$, dimensionless coordinate
τ time.

Mathematic symbols
J_0, J_1 first-kind zero- and first-order Bessel functions
$\operatorname{erfc} u = 1-\operatorname{erf} u = 1-2\int_0^u \exp(-\tilde{u}^2)\,d\tilde{u}/\sqrt{(\pi)}$ modified integral of probability.

whence the unknown velocity of the migration of isotherms is

$$v = \left[\frac{\partial(l_0-x)}{\partial\tau}\right]_T = \sum_{n=0}^{\infty} A'_{2n}(\tau)x^{2n}/2\sum_{n=0}^{\infty} nA_{2n}(\tau)x^{2n-1}. \quad (3)$$

It is possible to determine $A_{2n}(\tau)$ and $A'_{2n}(\tau)$ by substituting series (2) into equation (1)

$$C\sum_{n=0}^{\infty} A'_{2n}(\tau)x^{2n} = \lambda\sum_{n=1}^{\infty} 2n(2n+m-2)A_{2n}(\tau)x^{2n-2}$$

and equating the coefficient-functions $A'_{2n}(\tau)$ and $A_{2n}(\tau)$ of the left- and right-hand sides of this equation at the same powers of x.

In this case we obtain first

$$A_2(\tau) = CA'_0(\tau)/2m\lambda = A'_0(\tau)/2ma \quad (4)$$

and then, using equation (4),

$$A_4(\tau) = A''_0(\tau)/8(m+2)ma^2,$$

$$A_6(\tau) = A'''_0(\tau)/48(m+4)(m+2)ma^3$$

etc.

As a result, it is not difficult to show that the velocity of the migration of isotherms, determined by equation (3), is

$$v = \frac{ma}{x} \times \frac{A'_0(\tau)+\sum_{n=2}^{\infty} a^{-n+1}A_0^{(n)}(\tau)x^{2n-2}/S_1(m,n-1)}{A'_0(\tau)+\sum_{n=2}^{\infty} a^{-n+1}A_0^{(n)}(\tau)x^{2n-2}/S_2(m,n-1)}, \quad (5)$$

where

$$S_1(m,n-1) = \prod_{k=1}^{n-1} 2k(m+2k-2),$$

$$S_2(m,n-1) = \prod_{k=1}^{n-1} 2k(m+2k).$$

It follows from equation (2) that the function $A_0(\tau)$ coincides with the dependence of the temperature of the body centre of symmetry, $T(0,\tau)$, on time.

The analytical function $A_0(\tau)$ may be approximated by a power series. Then, in view of the fact that the partial sums of factorial series $S_1(m, n-1)$ and $S_2(m,n-1)$ grow with $n\to\infty$ faster than the partial sums of any power series the conclusion that in this case the series in the numerator and denominator of equation (5) everywhere converge according to D'Alambert is reached.

In fact, taking, for example, the dependence of the temperature of the body centre of symmetry as an exponential function of time $A_0(\tau)\sim\exp(\alpha\tau)$ we obtain a particular case of the series in equation (5)

$$a\sum_{n=2}^{\infty}\exp(\alpha\tau)(\alpha x^2/a)^n/S_1(m,n-1)x^2$$

and

$$a\sum_{n=2}^{\infty}\exp(\alpha\tau)(\alpha x^2/a)^n/S_2(m,n-1)x^2.$$

Calculating for the first of the series the limit of the nth to $(n-1)$th terms ratio, we have for $n\to\infty$

$$\lim_{n\to\infty}\frac{|\alpha x^2/a|}{2(n-1)(m+2n-4)} = 0$$

i.e. the mentioned series really converges everywhere by x. An analogous conclusion also refers to the second series considered.

Allowing for the fact that the case of heat propagation with infinitely high velocity is analysed, it is assumed that for $\tau > 0$ $A_0(\tau) \neq 0$, so the conclusion on the existence of v determined from equation (5) is derived.

Formula (5) in the case of linear time–variation of the temperature of the body centre of symmetry (quasi-stationary thermal conditions) $A_0(\tau) \sim \tau$ gives the relationship for determining the velocity of the migration of an isothermal surface

$$v = ma/x, \tag{6}$$

which is obtained also as a result of the analysis of familiar partial solutions [23] for constant heat flux density in the bounded body surface (the second-kind boundary condition) or as a result of the linear time–variation of temperature in it [24].

With the variation of the temperature of the symmetry centre over the decaying exponent (a regular heat mode)

$$A_0(\tau) = \beta \exp(-\alpha\tau), \quad \alpha > 0$$

based on equation (5) we come to the following relation for the calculation of the velocity of the migration of isotherms

$$v = \frac{ma}{x} \times \frac{1-\alpha x^2/2ma+\alpha^2 x^4/8(m+2)ma^2-\cdots}{1-\alpha x^2/2(m+2)a+\alpha^2 x^4/8(m+4)(m+2)a^2-\cdots}. \tag{5'}$$

Having introduced the variable $y = x\sqrt{(\alpha/a)}$ and making use of the known expansions of the function into Taylor's series, transform equation (5′) as

(a) at $m = 1$

$$v = \frac{a}{x} \times \frac{\sum_{n=1}^{\infty} (-1)^n y^{2n-2}/(2n-2)!}{\sum_{n=1}^{\infty} (-1)^n y^{2n-2}/(2n-1)!} = \frac{a}{x} \times \frac{-\cos y}{-\sin y/y} = ay \operatorname{ctg} y/x, \tag{5'_1}$$

(b) at $m = 2$

$$v = \frac{2a}{x} \times \frac{\sum_{n=0}^{\infty} (-1)^{n+1} (y/2)^{2n}/(n!)^2}{\sum_{n=0}^{\infty} (-1)^{n+1} (y/2)^{2n}/n!(n+1)!} = \frac{2a}{x} \times \frac{J_0(y)}{2J_1(y)/y}, \tag{5'_2}$$

(c) at $m = 3$

$$v = \frac{3a}{x} \times \frac{\sum_{n=1}^{\infty} (-1)^n y^{2n-2}/(2n-1)!}{3\sum_{n=1}^{\infty} y^{2n-2}/(2n-1)!(2n+1)} = \frac{3a}{x} \times \frac{-\sin y/y}{3(y\cos y-\sin y)^3/y} = -\frac{ay^2 \sin y}{x(y\cos y-\sin y)}. \tag{5'_3}$$

It is not difficult to show that at $\alpha = a\mu_1^2/l_0^2$ obtained from equations (5′₁), (5′₂), (5′₃) the following dimensionless velocities of the migration of isotherms exist

$$\tilde{v} = vl_0/a = \left[\frac{\partial(1-\xi)}{\partial Fo}\right]_T:$$

(a) $m = 1$

$$\tilde{v} = \mu_1 \operatorname{ctg} \mu_1\xi, \tag{5''_1}$$

(b) $m = 2$

$$\tilde{v} = \mu_1 J_0(\mu_1\xi)/J_1(\mu_1\xi), \tag{5''_2}$$

(c) $m = 3$

$$\tilde{v} = \mu_1^2/(\xi^{-1}-\mu_1 \operatorname{ctg} \mu_1\xi), \tag{5''_3}$$

where μ_1 is the first positive root of characteristic equations of the problem of non-stationary heat conduction with the boundary conditions of the third-kind at constant Bi that with a symetrically developing temperature field have the form

(a) $m = 1$

$$\mu = Bi/\operatorname{ctg} \mu, \tag{7_1}$$

(b) $m = 2$

$$\mu = BiJ_0(\mu)/J_1(\mu), \tag{7_2}$$

(c) $m = 3$

$$\mu = (1-Bi)/\operatorname{ctg} \mu. \tag{7_3}$$

The same formulae are also obtained with the corresponding consideration of the relation

$$\theta = \frac{T(x,\tau)-T_0}{T_f-T_0} = 1-A_{1,m}\chi_{1,m}(\xi)\psi_1(Fo), \tag{8}$$

describing a symmetrical one-dimensional temperature field at the stage of a regular heat mode. In equation (8) $A_{1,m}$ is the first thermal amplitude which, in the case of the third-kind boundary conditions, is equal to $A_{1,1} = 2/\mu_1$, $A_{1,2} = 2\mu_1 J_1(\mu_1)$, $A_{1,3} = 2/\mu_1$; $\chi_{1,m}(\xi)$ is the coordinate function of the form

$$\chi_{1,1} = \cos \mu_1\xi, \quad \chi_{1,2} = J_0(\mu_1\xi), \quad \chi_{1,3} = \sin \mu_1\xi/\xi \tag{9}$$

and the time function

$$\psi_1(Fo) = \exp(-\mu_1^2 Fo).$$

In fact, taking logs of equation (8) and then differentiating both sides with respect to Fo at $\theta = idem$, the equation shown is obtained

$$\tilde{v} = \left[\frac{\partial(1-\xi)}{\partial Fo}\right]_\theta = -\mu_1^2 \frac{\chi_{1,m}(\xi)}{\chi'_{1,m}(\xi)} \tag{10}$$

based on equation (9), particular cases ($5''_1$)–($5''_3$) as well.

Figure 1 illustrates, based on the corresponding computer calculation, the course of the lines $\theta = idem$ in the coordinates $(1-\xi)-Fo$ (the passed length–time) at $Bi = \infty$ for a plate. This figure indicates the fact that equidistant portions of the lines correspond to the period of regular thermal conditions describing the course of different isotherms $\theta = idem$ (shown by a dashed line).

Using the known relations for the description of temperature fields in the bodies with constantly operating uniformly spaced heat sources [23] as a basis and considering $T(x, \tau)$ from the non-equilibrium temperature of a stationary state we find also that in this case the velocities of the migration of isotherms $\tilde{v}$ on the stage of regularized thermal kinetics are calculated by equations ($5''_1$)–($5''_3$) and are independent of the power of the heat sources. It is also not difficult to prove that under the effect of an instantaneous heat source symmetrically located in a body the relationships ($5''_1$)–($5''_3$) are preserved, i.e. the value of $\tilde{v}$ is dependent in this case neither on the power of an instantaneous heat source nor on its position in a body.

Thus, the following common feature of thermal kinetics regularization may be introduced [7]: in bounded bodies with constant thermophysical properties the dimensionless velocity of the migration of isothermal surfaces in the periods of regular and quasi-stationary thermal conditions depends only on the coordinate of the body point and the kind of boundary conditions (the initial condition $T_0 = $ const. may be omitted in the majority of cases).

The formulated feature of thermal kinetics regularization is independent, along with the familiar characteristics of Dulnev and Kondratiev, of regularization by temperature fields [25] and, with the

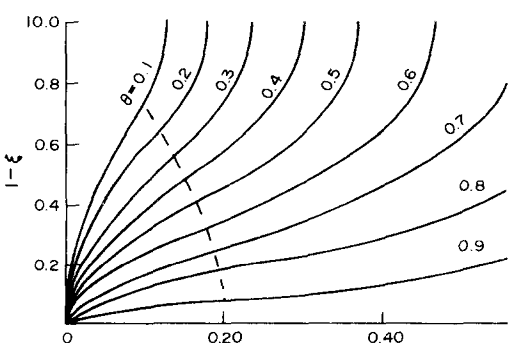

FIG. 1. Isotherms in an unbounded plate with $Bi \to \infty$.

Luikov principle, of regularization by heat fluxes that generalize the above ones [23].

3. SPECIFIC FEATURES OF THE MIGRATION OF ISOTHERMAL SURFACES IN CENTRAL PORTIONS OF BOUNDED BODIES

Consideration of correlation (5) for the determination of the velocity of the migration of isotherms indicates that the assumption on the boundedness of the derivatives $A_0^{(n)}(\tau)$ higher than the first-order of the body centre temperature with respect to time and on the smallness of $x(x \to 0)$ gives the equation

$$v = ma/x. \tag{11}$$

Thus, it turns out in these assumptions that the velocity v in central parts of the bounded bodies is independent of the character of the thermal conditions on the bounding surfaces.

In fact, equation (10) fully coincides with relation (6) for the parts of the body participating in quasi-stationary thermal conditions when the temperature of the body symmetry centre changes with time by a linear law that is provided by the case of the second-kind boundary conditions.

Now to reveal whether relation (11) is valid for the stage of regular thermal conditions when third-kind boundary conditions are assigned on the body surface, and what is the length of the body central part in which this relation is fulfilled.

In the obtained relations ($5''_1$)–($5''_3$) and in formula (10), correlating them, for regular thermal conditions, the first root of the characteristic equations (7_1)–(7_3) depends on the value of the Biot number, Bi, and, therefore, one may speak of the fact that the velocity $\tilde{v}$ is determined, to a certain extent, also by the thermal conditions on the bounding surface of the three simplest bodies (plate, cylinder, sphere) under consideration. To reveal the character of this relation, computer calculations of the values of $\tilde{v}$ at different Bi were made; the graphic presentation of the calculation results for a plate and a sphere with $Bi \to \infty$ and $Bi = 0.01$ are given in Fig. 2 (lines a, b correspond to $Bi \to \infty$ and $Bi = 0.01$ for a plate, lines c, d correspond to $Bi \to \infty$ and $Bi = 0.01$ for a sphere).

The most interesting result of the calculations is the fact that the velocity $\tilde{v}$ is practically independent of the value of the Biot number, Bi, that is valid, as is seen from Fig. 2, for central parts of bounded bodies $0 \leqslant \xi \leqslant 0.25$.

The last indicates the independence of $\tilde{v}$ in the mentioned region of the bodies of the thermal conditions on their outer surface and may be proved analytically.

It is known that μ_1 is always finite—$\mu_2 \leqslant \pi$. Then, for such small ξs (for small $\mu_1\xi$), that $\text{tg}\,\mu_1\xi \approx \mu_1\xi$, we have for an unbounded plate instead of equation ($5''_1$)

$$\tilde{v} = \mu_1 \,\text{ctg}\, \mu_1\xi \cong 1/\xi. \tag{11$_1$}$$

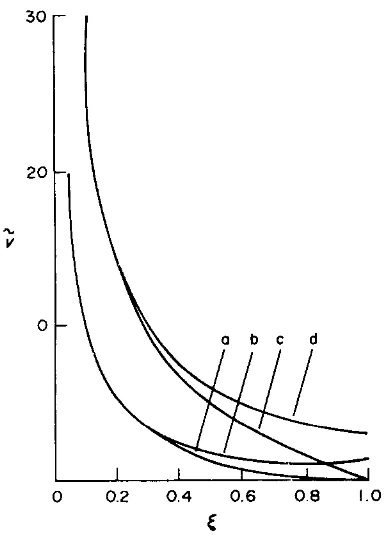

FIG. 2. Velocity of isotherms migration in a plate (lines a, b) and in a sphere (lines c, d) at $Bi = 0.01$ (lines a, c) and $Bi \to \infty$ (lines b, d).

The velocity $\tilde{v}$ in the central section of a cylinder, when $\mu_1\xi \ll 1$, may be calculated proceeding from the fact that in this case the first-kind Bessel function J_ν for $\nu > 0$ is represented as [26]

$$J_\nu(\mu_1\xi) \cong \Gamma(\nu+1)(\mu_1\xi/2)^\nu$$

(Γ is the gamma function), so that in equation ($5_2''$) for $\tilde{v}$ the following simplifications

$$\tilde{v} = \mu_1 J_0(\mu_1\xi)/J_1(\mu_1\xi)$$

$$\cong \mu_1(\mu_1\xi/2)^0\Gamma^{-1}(1)/(\mu_1\xi/2)^1\Gamma^{-1}(2) = 2/\xi \quad (11_2)$$

are obtained. Velocity $\tilde{v}$ in the sphere central portion may be calculated based on equation ($5_3''$)

$$\tilde{v} = \mu_1^2/(\xi^{-1} - \mu_1 \operatorname{ctg} \mu_1\xi)$$

$$= \mu_1/(1/\mu_1\xi - \cos\mu_1\xi/\sin\mu_1\xi)$$

and on the known Taylor series expansion of sine and cosine functions

$$\sin\mu_1\xi = \mu_1\xi - (\mu_1\xi)^3/3! + [(\mu_1\xi)^5/5!]\varphi_1(\mu_1\xi),$$

$$\cos\mu_1\xi = 1 - (\mu_1\xi)^2/2! + [(\mu_1\xi)^4/4!]\varphi_2(\mu_1\xi).$$

Then we have in the denominator of equation ($5_3''$)

$$\frac{1}{\mu_1\xi} - \frac{\cos\mu_1\xi}{\sin\mu_1\xi} = \frac{\mu_1\xi}{3}\varphi_3(\mu_1\xi)$$

where

$$\varphi_3(\xi) = \frac{1+3(\mu_1\xi)^2\varphi_1/5! - (\mu_1\xi)^2\varphi_2/4!}{1-(\mu_1\xi)^2/3! + (\mu_1\xi)^4\varphi_1/5!}.$$

Note that the functions $\varphi_1(\mu_1\xi)$, $\varphi_2(\mu_1\xi)$, $\varphi_3(\mu_1\xi)$ are analytical and $\varphi_1(0) = \varphi_2(0) = \varphi_3(0) = 1$.

As a result in the central portion of a sphere (when $\xi \ll 1$ or $\mu_1\xi \to 0$) we have

$$\tilde{v} \cong \mu_1/(\mu_1\xi/3) \cong 3/\xi. \quad (11_3)$$

The independence of $\tilde{v}$ is determined according to equations ($5_1''$)–($5_3''$) of the thermal conditions on the outer boundary of the body follows also from the easily proved relationship

$$\tilde{v}'_{\mu_1} = 0$$

for the central parts of a body, when $\mu_1\xi \to 0$.

It should be, of course, noted that under the regular thermal conditions, when the body temperature time variation is described by a simple exponent, the allowance for relationships (11_1)–(11_3) leads to the conclusion on the elliptical temperature distribution along the coordinate that for fixed *Fo* when heated has the form

$$\theta = 1 - F(m, \mu_1)(1 - \mu_1^2\xi^2/m)^{1/2}. \quad (12)$$

In fact, equations (11_1)–(11_3) with the exponential dependence of temperature on time may be obtained for small ξ or $\mu_1\xi$ only with the following description of the temperature field

$$\theta = 1 - A_{1,m}\exp(-\mu_1^2 Fo)(1 - \mu_1^2\xi^2/m)^{1/2}. \quad (13)$$

Actually, taking logs and differentiating equation (13) with respect to *Fo* we obtain the equation for the velocity $\tilde{v}$ in the form

$$\tilde{v} = m(1 - \mu_1^2\xi^2/m)/\xi \quad (14)$$

whence at small $\mu_1\xi$ there follows the discussed relationships (11_1)–(11_3). For an unbounded plate ($m = 1$) equation (12) can be easily obtained from the analysis of the known solution [23] and for a cylinder and sphere—only using equations (11_2), (11_3). The ellipse with the half-axes $1-\theta = F(m, \mu_1)$ and $\xi = \sqrt{(m)}/\mu_1$ is the graphic representation of relationship (12).

At small values of the Biot number ($Bi \leqslant 0.1$) the temperature distribution both in the central portions and in the entire body is described by the elliptical law (12) at the regular stage.

The above results obtained on the specific features of bodies may be with even greater ground when related to the cores of their components which find wide application in engineering and thermal physics. In the latter case they are used when thermal conductivity λ is determined by the method of bicalorimeters of a regular mode (plane, cylindrical, spherical) as well as in devices for dynamic determination of the thermophysical properties of bodies. It is shown in ref. [9] that the application of relationship (11) allows the determination of the thermal diffusivity of the core material with no limitations on the core–shell pair in the so-called thermally insulated cores when $\tilde{v}$ is independent either of the shell properties or the thermal conditions at the bicalorimeter boundary, etc. The latter greatly simplifies a thermophysical experiment.

4. SPECIFIC FEATURES OF THE MIGRATION OF ISOTHERMAL SURFACES ABOUT THE OUTER SURFACE OF A BODY AT THE STAGE OF REGULAR THERMAL MODE

In Section 3 the independence of $\tilde{v}$ in the central portion of a body at the stage of a regular thermal mode of the thermal conditions on its bounding surface is shown. Naturally, a problem arises with respect to the dependence of $\tilde{v}$ at this surface of a body on the thermal conditions on the surface when the region of regularized kinetics spreads over the entire body.

It is not difficult to see that at $\xi = 1$ (on the surface flown around) equations $(5_1'')$–$(5_3'')$ became as follows

$$\tilde{v} = \mu_1 \operatorname{ctg} \mu_1, \qquad (15_1)$$

$$\tilde{v} = \mu_1 J_0(\mu_1)/J_1(\mu_1), \qquad (15_2)$$

$$\tilde{v} = \mu_1^2/(1-\mu_1 \operatorname{ctg} \mu_1). \qquad (15_3)$$

With allowance for the characteristic equations (7_1)–(7_3) relations (15_1)–(15_3) may be transformed to the form valid for all of the three bodies under consideration

$$\tilde{v} = \mu_1^2/Bi. \qquad (16)$$

In Fig. 3 the graph of the dependence of $\tilde{v} = \mu_1^2/Bi$ for a plate, cylinder and sphere (lines a, b and c, respectively) is plotted that is constructed using the data obtained on a computer. The analysis of Fig. 3 leads to the conclusion that $\tilde{v}$ is strongly dependent on the intensity of convective heat transfer at small Biot numbers; with growing Bi ($Bi \geqslant 30$) the mentioned relation becomes weak.

It is easy to see that relationship (16) may form the basis for a technique to determine the convective heat transfer coefficient α. In fact, interpreting the results of the corresponding thermophysical experiment as migration of isothermal surfaces in a heat sensing element we obtain a constant velocity of the migration of isothermal surfaces $\tilde{v}$ at a body bounding surface $\xi = 1$ when the entire body is involved in the stage of regular thermal mode, and then using a computer and based on relations (7_1)–(7_3), (16) or, in case the required accuracy of α determination is not high, we, resorting to Fig. 3, find the unknown Biot number $Bi = \alpha l_0/\lambda$. It is clear that in such a way the coefficient of convective heat transfer α, that is constant in time, may be determined. However, this method does not

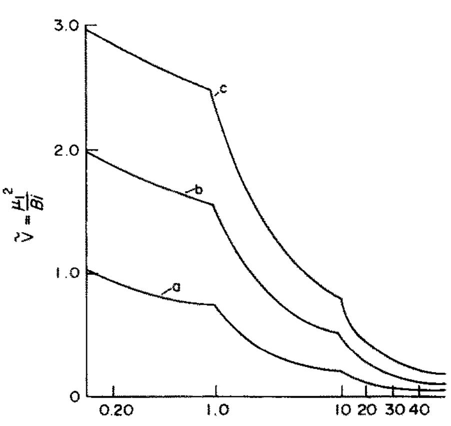

FIG. 3. Velocity of isotherms migration near a surface of a plate, cylinder and sphere (lines a, b, c, respectively).

require knowledge of the surrounding temperature and this is the obvious simplicity of this calculation technique, and may be decisive for its practical implementation. As has been found earlier, there is no possibility of determining the value of α by one heat sensing device when not knowing the absolute or excess value of the temperature of the liquid or gas flow. The technique described is very important for the determination of the coefficient of convective heat transfer from reacting or two phase flows, in the case of small cross-sections, etc.

It is interesting to note that when the entire body is involved in the stage of the regular thermal mode the notion of equivalent thermal resistance of convective heat transfer in the process of non-stationary heat conduction may be introduced based on the consideration of successive motion of isothermal surfaces in a body. Earlier this notion existed only for the process of stationary heat conduction. At the known value of this resistance the assignment of the third-kind boundary conditions may be substituted by the first-kind boundary conditions, which is of interest for both the simplification of analogue modelling of non-stationary processes and the corresponding calculational technique. It has been shown that this substitution is possible, at least, for the final stage of the considered process.

The dimensionless time of isotherm migration $\theta = idem$ on the elementary path $d(1-\xi)$

$$dFo = d(1-\xi)/\tilde{v} = -d\xi/\tilde{v}$$

and the final time of migration $\theta = idem$ from the point with the coordinate ξ in a body up to its symmetry centre $\xi = 0$ is

$$Fo = \int_{\xi}^{0} d\xi/\tilde{v}.$$

At the stage of a regular thermal mode with allowance for relationships $(5_1'')$–$(5_3'')$ for $\tilde{v}$ obtained, respectively, for a plate, cylinder, and sphere

$$Fo = \ln(\cos \mu_1 \xi)/\mu_1^2, \qquad (17_1)$$

$$Fo = \ln J_0(\mu_1 \xi)/\mu_1^2, \qquad (17_2)$$

$$Fo = \ln(\sin \mu_1 \xi/\xi)/\mu_1^2. \qquad (17_3)$$

For the first-kind boundary conditions, when the equivalent thermal resistance of convective heat transfer during the entire process of non-stationary heat conduction is equal to zero, the values of μ_1 are equal to $\pi/2$ for a plate, 2.4048 for a cylinder and π for a sphere. Substitution of these values of μ_1 into equations (17_1)–(17_3) and $\xi = 1$ into the integration limits, when the entire body is involved in the stage of the regular thermal mode, yields $Fo = \infty$. This fact corresponds to the infinite time of the body heating up to the surrounding temperature ($\theta = 1$). In this case an isothermal surface, $\theta = 1$, moves in the region of a regular thermal mode virtually all the time.

Then, in a more general case of the third-kind boundary conditions, obtained on the basis of equations (17_1)–(17_3) the value of $Fo \to \infty$ for some $\xi > 1$, i.e. with the solid body 'building-up' by a material with an equivalent thermal resistance to convective heat transfer in a non-stationary process.

Based on equations (17_1)–(17_3) the value of ξ found that provides $Fo \to \infty$ at any μ_1 for a plate, cylinder and sphere from the relationships

$$\mu_1 \xi = \pi/2, \quad (18_1)$$

$$\mu_1 \xi \cong 2.4048, \quad (18_2)$$

$$\mu_1 \xi = \pi \quad (18_3)$$

whence new sizes of bodies mentioned (after additional layer 'building-up') are obtained

$$\xi = \pi/2\mu_1, \quad \xi \cong 2.4048/\mu_1, \quad \xi = \pi/\mu_1. \quad (18')$$

Thus, consideration of the final stage of the process of non-stationary heat conduction in a body with the characteristic size l_0 under the third-kind boundary conditions may be substituted by consideration of the process under the first-kind boundary conditions in a body with the characteristic size $l_0\pi/2\mu_1$, $2.4048 l_0/\mu_1$, $l_0\pi/\mu_1$. The applicability region of this substitution was found on the basis of the corresponding computer calculations: within the range $Bi \in [5, \infty)$ this substitution does not distort the picture of the migration of isothermal surfaces $\theta \in [0.1; 1.0]$, when $Bi \in [0.004; 1.0]$ this substitution allows the admissible accuracy only for isothermal surfaces $\theta \in [0.9; 1.0]$, i.e. only for the final stage of body heating.

5. REGULARITIES OF THE MIGRATION OF ISOTHERMAL SURFACES UNDER NON-LINEAR BOUNDARY CONDITIONS

Radiative heat transfer is characterized by nonlinearity in the boundary condition for equation (1) which at constant thermophysical properties of the material in the dimensionless variables has the form

$$-\left.\frac{\partial \vartheta}{\partial \xi}\right|_{\xi=1} = Sk(\vartheta^4|_{\xi=1} - 1).$$

Unfortunately, literature has no voluminous data on the structure of the temperature fields for this case which could be sufficient for the determination of the regularities of the migration of isothermal surfaces since they are given only for the characteristic points of a body [27]. Therefore, the corresponding problem on the non-stationary symmetric temperature field in a plate with identical initial dimensionless temperature $\vartheta_0 = T_0/T_r$ equal to 0; 0.166, 0.200, 0.333, 0.500, 0.800 at the Stark number, Sk, values: 0.01, 0.02, 0.05, 0.10, 0.20, 0.50, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0 was solved on a computer by the grid method using the implicit absolutely stable Laasonen scheme [28]. The space step $\Delta\xi = 0.1$ was taken in the calculation; the dimensionless relative temperature $\theta = [T(\xi, Fo) - T_0]/(T_r - T_0)$ was determined at the plate points with the coordinates ξ equal to 1.00, 0.95, 0.85, 0.75, 0.65, 0.55, 0.45, 0.35, 0.25, 0.15, 0.05 at the time step $Fo = 0.02$. The coincidence of the calculated values with the data of ref. [27] obtained by the method of analogue modelling was virtually complete.

During calculations the subprogramme 'Interpretation' provided printing of the location at the time of the prescribed temperature $\theta = idem$ thus making it possible both to calculate the rate of isothermal surface migration and to graphically interpret the course of isothermal surfaces in the coordinates $(1-\xi) - Fo$. Figures 4 and 5 show this interpretation for $\vartheta_0 = 0.5$ at $Sk = 0.2$ and $Sk = 1.0$, respectively.

The results of calculations indicate the onset of the stage of regularized thermal kinetics with time-unchangeable local velocity of the migration of isothermal surfaces in the body $\theta = idem$ which is confirmed by the presence of equidistant portions of the lines of different $\theta = idem$ in Figs. 4 and 5. The region of regularized thermal kinetics is first formed in the central plate sections and then it expands with time. Steady-state thermal kinetics under the boundary conditions of the radiation law at T_r = const. is, first, a quasi-stationary thermal regime which then changes over to a regular thermal regime of the first kind. In this case in the central plate section, $0 \leqslant \xi < 0.25$, the velocity of the migration of isothermal surfaces coincides with that determined by equation (11_1)

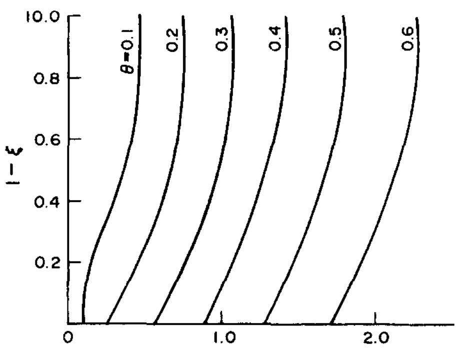

FIG. 4. Isotherms in an unbounded radiation-heated plate at $\vartheta_0 = 0.5$ and $Sk = 0.2$.

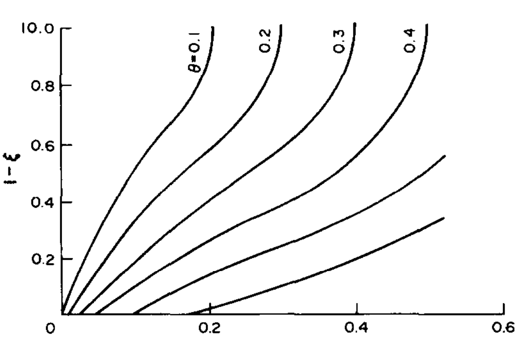

FIG. 5. Isotherms in an unbounded radiation-heated plate at $\vartheta_0 = 0.5$ and $Sk = 1.0$.

$$\tilde{t} = 1/\xi.$$

The laws of heat kinetics regularization found from computer calculations are important for determining the thermophysical properties of the body material since realization of boundary conditions by the radiation law is simple and it is often used in research work.

In fact, measuring the velocity of migration of the isothermal surfaces v in the plate central section, $0 \leqslant \xi \leqslant 0.25$, we find thermal diffusivity by the equation

$$a = v \times x = \left[\frac{\partial(l_0 - x)}{\partial \tau}\right]_a \times x. \qquad (11')$$

The use of this equation does not require knowledge of the temperature of a radiation T_r or of heat fluxes and it is not limited by any conditions imposed on their values, rate of variation, etc.

The obtained laws of the migration of isothermal surfaces under non-linear boundary conditions by the radiation law may also form the basis for improving the calculations of temperature fields, as is shown in ref. [10].

It should be emphasized that the considered radiative heating of a plate at T_r = const. is a monotonous process. Consequently, in any process of monotonous heating (cooling) of a body under non-linear boundary conditions regularization of heat kinetics takes place with the fixed regularity and time-invariable local velocity of the migration of isothermal surfaces, at least in the central portions of bounded bodies. This conclusion is in full conformity with the results obtained in Sections 2 and 3.

6. THE LAWS OF THE MIGRATION OF ISOTHERMAL SURFACES IN BOUNDED BODIES WITH TEMPERATURE-DEPENDENT THERMOPHYSICAL PROPERTIES

The consideration of the above-mentioned case is very important for thermophysical practice. Here a non-stationary one-dimensional temperature field in the bodies of the simplest configuration—plate ($m = 1$), cylinder ($m = 2$), sphere ($m = 3$)—is found from the solution of the differential equation

$$C(T)\frac{\partial T}{\partial \tau} = \frac{1}{x^{m-1}}\frac{\partial}{\partial x}\left[x^{m-1}\lambda(T)\frac{\partial T}{\partial x}\right],$$

$$\tau > 0, \quad 0 < x < l_0 \qquad (19)$$

with the corresponding boundary conditions.

Application of the technique formulated in Section 2 to the presentation of the unknown solution in the form of equation (2)

$$T(x, \tau) = \sum_{n=0}^{\infty} A_{2n}(\tau)x^{2n}$$

gives the function $A_{2n}(\tau)$ in the form

$$A_2(\tau) = A'_0(\tau)/2ma(T). \qquad (20)$$

$$A_4(\tau) = \frac{A'_2(\tau)}{4a(T)(m+2)} + \frac{\mathrm{d}C(T)}{\mathrm{d}T} \times \frac{A_2(\tau)A'_0(\tau)}{4\lambda(T)(m+2)} \qquad (21)$$

or

$$A_4(\tau) = \frac{A'_2(\tau)}{4a(T)(m+2)} + \frac{mA_2^2(\tau)}{2(m+2)}\frac{\mathrm{d}\ln C(T)}{\mathrm{d}T} - \tfrac{1}{2}A_2^2(\tau)\frac{\mathrm{d}\ln\lambda(T)}{\mathrm{d}T}. \qquad (22)$$

$A_6(\tau)$ and $A_8(\tau)$ are calculated analogously.

It must be emphasized that in the expressions obtained for $A_{2n}(\tau)$ the body material thermophysical properties a, λ, c and their derivatives with respect to T are found at the temperature of the body symmetry centre $A_0(\tau)$ at the given time instant τ.

In the case of a quasi-linear equation (19) in formula (3) for the determination of the velocity of the migration of isotherms

$$v = \sum_{n=0}^{\infty} A'_{2n}(\tau)x^{2n}/2\sum_{n=1}^{\infty} nA_{2n}(\tau)x^{2n-1}$$

it is difficult to judge, without additional assumptions about $A_{2n}(\tau)$ and $A'_{2n}(\tau)$, the convergence of the denominator and numerator.

At small x, assuming $A_{2n}(\tau)$ and $A'_{2n}(\tau)$ (and thus $\mathrm{d}\ln C(T)/\mathrm{d}T$ and $\mathrm{d}\ln\lambda(T)/\mathrm{d}T$) to be limited, we obtain

$$v \cong [A'_0(\tau) + x^2A'_2(\tau)]/[2xA_2(\tau) + 4x^3A_4(\tau)] \qquad (3')$$

or

$$v \cong A'_0(\tau)/2xA_2(\tau). \qquad (3'')$$

With allowance for equation (20) for $A_2(\tau)$ relation (3″) changes to

$$v \cong \frac{ma(T)}{x} \qquad (3''')$$

where $a(T)$ is the thermal diffusivity of the body material at the temperature of its centre at the time instant τ when velocity v is calculated at the point x near the body symmetry centre.

Thus, it is proved, in particular, that in the central portion of bounded bodies the velocity of the migration of isothermal surfaces depends on the thermal diffusivity $a(T) = \lambda(T)/C(T)$ rather than on $C(T)$ and $\lambda(T)$ separately.

The latter may be also revealed in the analysis of equation (19) in the range of small ξ.

Relation (3‴) forms a theoretical basis for the technique of determination of thermal diffusivity of body materials $a(T)$ during one test without any restrictions on the rate of temperature field variation in a body and, consequently, to the form of the boundary conditions.

7. MIGRATION OF ISOTHERMAL SURFACES ON THE PERIPHERY OF BOUNDED BODIES

It is generally known that use of operational calculus based on Laplace transforms with respect to time variable allowed Luikov to show that at small times temperature variations on the periphery of bounded bodies with constant thermophysical properties correspond to regularities of semi-bounded bodies [23]. It is natural that the velocity of the migration of isothermal surfaces in bounded bodies will coincide in this case with that for semi-bounded bodies.

Thus, in a particular case of a half-space with the same initial temperature and time–constant temperature on a bounding surface obtain, based on ref. [23], the equation of the migration of isotherms

$$v = \left(\frac{\partial x}{\partial \tau}\right)_\theta = \frac{1}{2}\frac{x}{\tau} = 2a\Phi^2(\theta)/x \tag{23}$$

where $\Phi(\theta) = \mathrm{erfc}^{-1}\theta$ is the function inverse to the modified probability integral erfc u.

It follows from equation (23) that lines $\theta = idem$ in the coordinates x–τ (path–time) corresponds to the upper branch of the square parabola.

In the coordinates x–$\sqrt{(a\tau)}$ the course $\theta = idem$ is obtained in the form of straight lines beginning from the coordinate origin at the angle to the abscissa axis whose tangent is equal to $2\Phi(\theta)$, because equation (23) yields

$$x^2 = 4a\tau\Phi^2(\theta)$$

and, proceeding from the physical sense of the process

$$x = 2\Phi(\theta)\sqrt{(a\tau)}. \tag{24}$$

To compare with a half-space, the corresponding results for an unbounded plate (and any other bounded body) with the first-kind boundary conditions should be interpreted in the coordinates $(l_0$–$x)$ or $(l_0$–$x)$–$\sqrt{(a\tau)}$ (in a dimensionless form, in the coordinates $(1$–$\xi)$–Fo or $(1$–$\xi)$–$\sqrt{(Fo)}$).

It is easy to see that in Fig. 1 for an unbounded plate with constant thermophysical properties at the same initial temperature and symmetric development of a temperature field of surface heating with the constant temperature the initial portions of isotherms $\theta = idem$ correspond to the upper branch of the square parabola in the coordinates $(1$–$\xi)$–$\sqrt{(Fo)}$ in Fig. 6.

The same coincidence is found in ref. [7] for a half-space and the periphery of an unbounded plate between the velocities of the migration of isotherms $\theta = idem$ and for the third-kind boundary conditions. Computer calculations show that at constant thermophysical properties the initial sections of the lines $\theta = idem$ in a plate to the moment when the entire body is involved in the stage of the regular thermal conditions up to the sections with the regularized kinetics of heating correspond to the thermal laws of a half-space. Figures 7 and 8 present the results

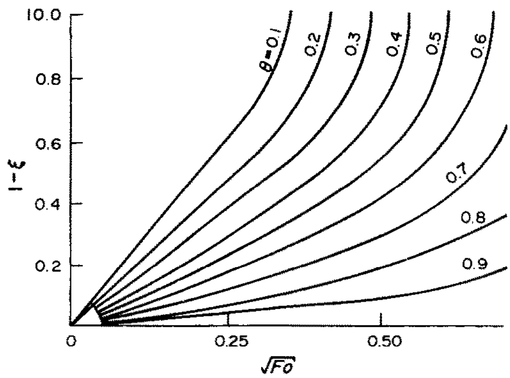

FIG. 6. Isotherms in an unbounded plate when $Bi \to \infty$.

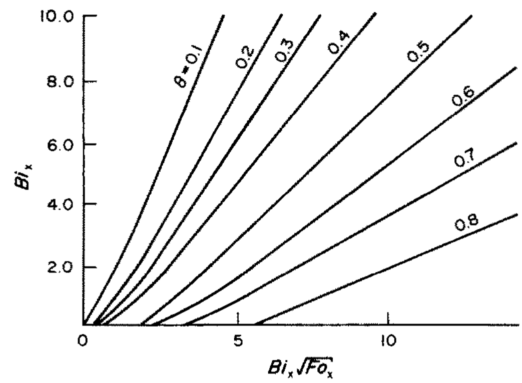

FIG. 7. Isotherms in a half-space at $Bi = 10$.

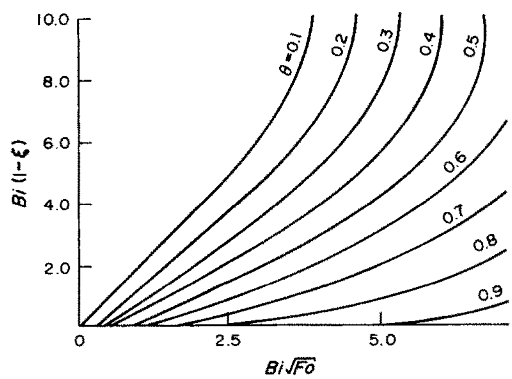

FIG. 8. Isotherms in an unbounded plate at $Bi = 10$.

of these calculations at $Bi = 10$ for a half-space in the coordinate system $Bi_x - Bi_x\sqrt{(Fo_x)}$ $(Bi_x = \alpha x/\lambda$, $Fo_x = a\tau/x^2)$ and for an unbounded plate in the diagram $Bi\,(1-\xi) - Bi\sqrt{(Fo)}(Bi = \alpha l_0/\lambda,\ Fo = a\tau/l_0^2)$.

The proof of the fulfilment of the 'half-space period' regularity for the temperature-dependent thermophysical properties of the body material was based, as well as in the case of a linear problem, on a numerical experiment. Here, using the Crank–Nicholson scheme, the problem of symmetrical development of a temperature field on an unbounded plate with the same dimensionless temperature $\theta(\xi, 0) = 0$ at time-constant temperature $\theta(\xi = 1, Fo) = 1$ on bounding

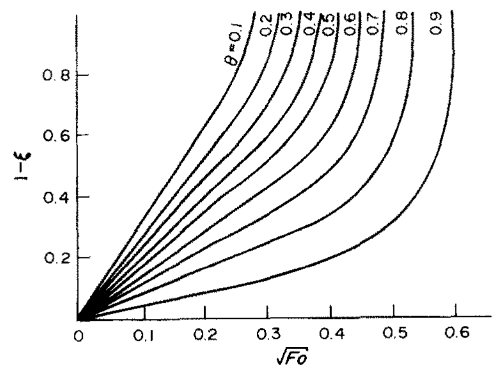

FIG. 9. Isotherms in an unbounded plate at $\tilde{a} = \tilde{\lambda}(\theta) = (1-\beta\theta)^2$ at $\beta = 0.50$.

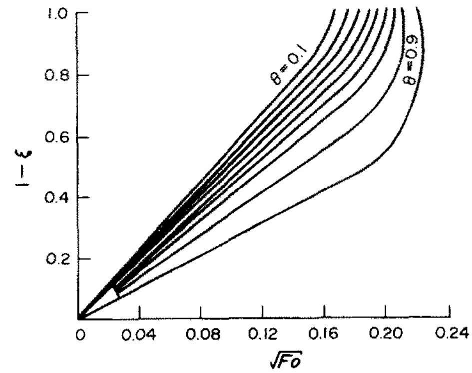

FIG. 10. Isotherms in an unbounded plate at $\tilde{a}(\theta) = \tilde{\lambda}(\theta) = (1-\beta\theta)^{-2}$ at $\beta = 0.90$.

surfaces was solved on a computer by the grid method. Omitting the details stated in ref. [15] it is shown that the volumetric heat capacity of the body material was assumed to be constant ($\tilde{C} = C/C_0 = 1$) and the thermal conductivity $\tilde{\lambda}(\theta) = \lambda(\theta)/\lambda_0$, and, consequently, the thermal diffusivity $\tilde{a}(\theta) = a(\theta)/a_0$ are dependent on θ (the values of λ_0, C_0, $a_0 = \lambda_0/C_0$ are taken at $\theta = 0$) as

$$\tilde{a}(\theta) = \tilde{\lambda}(\theta) = 1+\beta\theta \quad (25)$$

$$\tilde{a}(\theta) = \tilde{\lambda}(\theta) = \exp(\beta\theta) \quad (26)$$

$$\tilde{a}(\theta) = \tilde{\lambda}(\theta) = (1-\beta\theta)^{-2}. \quad (27)$$

Note that, according to the opinion of the majority of researchers, a real form of $\tilde{\lambda}(\theta)(\tilde{a}(\theta))$ for many materials is close to linear or experimental laws.

Besides the calculation of the values of $\theta(\xi, Fo)$ the program 'Interpretation' was created that provided the search of the prescribed values of θ at the fixed points of space on each time layer. The number of time layers, on which the mentioned values of θ were determined, are shown in Table 1. The correctness of the algorithm of the basic problem solution was confirmed by the test problem of Samarsky and Sobol [29] for a section with $\tilde{\lambda}(\theta) = \tilde{a}(\theta) = 0.5\theta^2$.

In Figs. 9 and 10 the graphs are presented of some computer solutions for a plate, when $\tilde{a}(\theta) = \tilde{\lambda}(\theta) = (1-\beta\theta)^{-2}$ at $\beta = 0.50$ and 0.90, in the coordinates $(1-\xi)$–$\sqrt{(Fo)}$, where $(1-\xi)$ is the length from the body outer surface, $Fo = a_0\tau/(l_0-x)^2$ is the Fourier number constructed using the value of thermal diffusivity at $\theta = 0$.

The results of computer calculations indicate the fact that in the periphery portions of a plate and in a half-space for which, in particular, with relation (27) there is the Fujita algorithm [26] for obtaining an accurate solution, the values of the isochrones $\zeta = (1-\xi)/2\sqrt{(Fo)}$ coincide for $\theta = idem$ (in Figs. 9 and 10 the tangent of the inclination angle of the rectilinear portions of the lines $\theta = idem$ coincides with the doubled value of ζ given in Table 1).

8. SPECIFIC FEATURES OF THE KINEMATICS OF TEMPERATURE FIELDS IN A HALF-SPACE WITH TEMPERATURE-DEPENDENT THERMOPHYSICAL CHARACTERISTICS

It was interesting to compare temperature fields in a half-space for different characters of the dependence of thermophysical properties on temperature. The fact, that at any $a = a(\theta)$, as is shown in refs. [23, 30], the values of isochrones $\zeta = x/2\sqrt{(a_0\tau)}$ are the arguments for θ, forms the basis for this comparison. It allows one to easily systematize the solutions of boundary value problems for a half-space, the exact solutions of which are found only for some forms of $a = a(\theta)$ by Fujita (in ref. [5]), Crank [31], Friedmann [32]. In Tables 2 and 3 the temperature fields in a half-space obtained by a computer are interpreted in the form of θ-to-ζ relations for monotonous relationships of thermal diffusivity

$$a(\theta) = a_0(1+\beta\theta), \quad (28)$$

$$a(\theta) = a_0\exp(\beta\theta), \quad (29)$$

$$a(\theta) = a_0(1-\beta\theta)^{-2}, \quad (30)$$

$$a(\theta) = a_0(1-\beta\theta)^{-1} \quad (31)$$

Table 1. The values of isochrones for $\tilde{a}(\theta) = \tilde{\lambda}(\theta) = (1-\beta\theta)^{-2}$ obtained by a computer

	Isochrones $\zeta = x/2\sqrt{(a_0\tau)}$ for θ								
β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.50	1.536	1.299	1.127	0.979	0.838	0.698	0.551	0.391	0.211
0.80	2.158	1.964	1.818	1.684	1.546	1.392	1.208	0.966	0.612
0.90	2.797	2.639	2.519	2.406	2.287	2.148	1.969	1.706	1.238
0.95	3.560	3.431	3.405	3.328	3.312	3.084	3.027	2.806	2.205

Table 2. The values of isochrones for $\bar{a}/a_0 = 1.5$

Temperature θ	Isochrones $\zeta = x/2\sqrt{(a_0\tau)}$ for $\tilde{a}(\theta)$: $\tilde{a} = 1$	$\tilde{a} = 1+\beta\theta$, $\beta = 1.00$	$\tilde{a} = (1-\beta\theta)^{-1}$, $\beta = 0.5828$	$\tilde{a} = (1-\beta\theta)^{-2}$, $\beta = 0.3333$	$\tilde{a} = \exp(\beta\theta)$, $\beta = 0.7626$
0.05	1.3895	1.6003	1.5724	1.6036	1.6081
0.1	1.1635	1.3858	1.3708	1.3613	1.3664
0.2	0.9020	1.1164	1.1061	1.1055	1.1082
0.3	0.7323	0.9469	0.9440	0.9481	0.9483
0.4	0.5940	0.7938	0.8024	0.7983	0.7964
0.5	0.4768	0.6556	0.6716	0.6662	0.6622
0.6	0.3709	0.5248	0.5477	0.5408	0.5349
0.7	0.2727	0.3961	0.4229	0.4143	0.4073
0.8	0.1794	0.2673	0.2934	0.2846	0.2777
0.9	0.0890	0.1360	0.1543	0.1478	0.1429
0.95	0.0444	0.0688	0.0795	0.0755	0.0726

Table 3. The values of isochrones for $\bar{a}/a_0 = 2$

Temperature θ	Isochrones $\zeta = x/2\sqrt{(a_0\tau)}$ for $\tilde{a}(\theta)$: $\tilde{a} = 1$	$\tilde{a} = 1+\beta\theta$, $\beta = 2$	$\tilde{a} = (1-\beta\theta)^{-1}$, $\beta = 0.8$	$\tilde{a} = (1-\beta\theta)^{-2}$, $\beta = 0.5$	$\tilde{a} = \exp(\beta\theta)$, $\beta = 1.2564$
0.05	1.3895	1.8015	1.7200	1.7200	1.7401
0.1	1.1635	1.5592	1.5227	1.5349	1.5481
0.2	0.9020	1.2827	1.2578	1.2684	1.2796
0.3	0.7323	1.120	1.1008	1.1050	1.1093
0.4	0.5940	0.9536	0.9723	0.9690	0.9654
0.5	0.4768	0.7954	0.8412	0.8303	0.8200
0.6	0.3709	0.6448	0.7109	0.6921	0.6753
0.7	0.2727	0.4917	0.5737	0.5482	0.5262
0.8	0.2794	0.3350	0.4209	0.3900	0.3668
0.9	0.0890	0.1719	0.2370	0.2108	0.1931
0.95	0.0444	0.0872	0.1279	0.1101	0.0994

at different values of $\bar{a}/a_0$ calculated as integral-mean relative thermal diffusivities within the range $\theta \in [0;\ 1]$ which are $1+\beta/2$, $(\exp\beta-1)/\beta$, $(1-\beta)^{-1}$, $-[\ln(1-\beta)]/\beta$ for equations (28)–(31), respectively.

The comparison of ζ at $\bar{a}/a_0 = idem$ in Tables 2 and 3 shows that within the range $1 \leqslant \bar{a}/a_0 \leqslant 1.5$ the arguments ζ for the quantities $\theta = idem - \theta \in [0.05;\ 0.60]$ at $\bar{a}/a_0 = idem$ do not differ greatly. Note that at the value $\bar{a}/a_0 = 1.5$ there correspond successfully maximum relative thermal diffusivities a_{max}/a_0 equal to 2.0; 2.14; 2.25; for equations (28)–(31). These variations of thermal diffusivity with the growth of temperature should be assumed to be very strong (for the cases corresponding to equations (28)–(31) they are shown graphically in Fig. 11), covering the behaviour of $\tilde{a}(\theta)$ for a considerable part of known materials.

Thus, for many of the materials applied in practice, the thermophysical properties of which are described by relations (28) and (29), the temperature field of a half-space is determined, with acceptable accuracy, by the familiar exact solution of the corresponding problem with functions (30), (31) at $\bar{a}/a_0 = idem$ $(1 \leqslant a/a_0 \leqslant 1.5)$.

Analysis of Tables 2 and 3 indicates also high sensitivity of temperature field determination to the temperature relation $\tilde{a} = \tilde{a}(\theta)$ thus making the problem of finding the latter with great accuracy urgent.

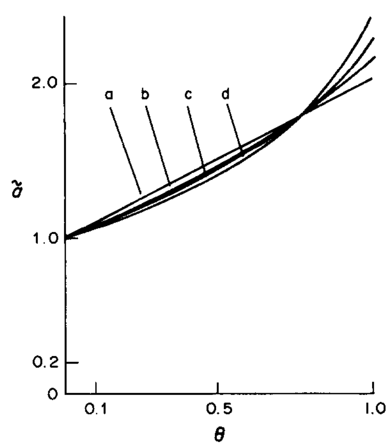

FIG. 11. Relations for $\tilde{a}(\theta)$: (a) $\tilde{a} = 1+\beta\theta$ $(\beta = 1)$; (b) $\tilde{a} = \exp(\beta\theta)$ $(\beta = 0.7626)$; (c) $\tilde{a} = (1-\beta\theta)^{-2}$ $(\beta = 0.3333)$; (d) $\tilde{a} = (1-\beta\theta)^{-1}$ $(\beta = 0.5828)$.

The results obtained led to the conclusion that under the first-kind boundary conditions the calculation of temperature fields on the periphery portions of bounded bodies as well may be substituted by the solution of a non-linear problem for 'identical' half-space when non-linear partial differential problems may be reduced to partial equations, whose solution is easier to be obtained, or when it is possible, in some cases, to make special substitutions, to come to linear problems [33, 34]. In those specific cases when $1 \leqslant \bar{a}/a_0 < 1.5$ one can find temperature fields on the periphery portions of an unbounded plate with linear and exponential relations (28), (29) for $a(\theta)$ using accurate solutions for the half-space with $a(\theta)$ determined according to (30) or (31).

It is clear that all of the above-obtained results are valid also for the case $\tilde{C}(\theta) \neq 1$ if $w = \int_0^\theta \tilde{C}(\theta)\, d\theta$ is taken instead of θ.

Thus, it may be confirmed that the temperature field is formed on the periphery portions of bounded bodies following the laws of an 'identical' (in the sense of coinciding thermophysical properties) half-space at the first-kind boundary conditions. In fact, according to (30) at $\beta = 0.9$ the value of $\tilde{\lambda}(\theta)[a(\theta)$ at $\tilde{C} = 1]$ grows from 1 at $\theta = 0$ to 100 at $\theta = 1$ and even in the case of such a strong variation of thermophysical properties the laws of a half-space are valid.

Moreover, in this case the laws of the 'period of a half-space' cover a great portion of a bounded body (Fig. 10).

9. CONCLUSION

The consideration of the process of non-stationary heat conduction in terms of the migration of isothermal surfaces allows one to find equations for determining the velocity of their migration in a half-space and in bounded bodies at constant and temperature-dependent thermophysical characteristics of a material. In the analysis of these formulae a new common feature of thermal kinetics regularization and specific features of isotherm migration in central portions of bounded bodies as well as near their outer surfaces are found that are important for thermophysical applications.

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METHODE DE LA MIGRATION DES ISOTHERMES DANS L'ETUDE DU TRANSFERT DE CHALEUR ET DE MASSE EN THEORIE ET EN PRATIQUE—I. CINEMATIQUE DES CHAMPS DE TEMPERATURE

Résumé—Le mécanisme de la conduction thermique variable dans les corps avec ou sans transition de phase de la substance est considéré en terme de migration des surfaces isothermes. Les équations sont données pour déterminer la vitesse de leur mouvement dans un demi-espace et dans les corps finis avec des propriétés thermophysiques dépendantes de la température. On trouve dans ce cas un nouveau comportement commun de la régularisation cinétique thermique qui est indépendant de la vitesse du mouvement des isothermes. L'indépendance dans les zones centrales des corps limités vis-à-vis de la situation thermique de leurs surfaces externes est aussi constatée. On considère l'effet des conditions aux limites non linéaires sur la cinétique des champs de température avec rayonnement. Les régularités et les traits spécifiques de la formation des champs de température constituent la base pour l'identification du coefficient de transfert thermique et pour la dépendance de la diffusivité thermique vis-à-vis de la température.

DAS VERFAHREN DER ISOTHERMENWANDERUNG IN THEORIE UND PRAXIS DER FORSCHUNG AUF DEM GEBIET DER WÄRMEÜBERTRAGUNG—I. KINEMATIK VON TEMPERATURFELDERN

Zusammenfassung—Der instationäre Wärmeleitprozeß in Körpern mit und ohne Phasenänderung der Substanz wird mit Hilfe von Ausdrücken für die Bewegung isothermer Oberflächen erfaßt. Die Gleichungen werden abgeleitet, um die Bewegungsgeschwindigkeit der Isothermen innerhalb eines Halbraumes und begrenzter Körper zu bestimmen. In diesem Fall wird eine neue allgemeine Eigenschaft thermodynamischer Gesetzmäßigkeiten gefunden, welche unabhängig von der Fortpflanzungsgeschwindigkeit der Isothermen bezüglich der Zeit ist. Weiterhin wird auch die Unabhängigkeit dieser Größe im Inneren begrenzter Körper von der thermischen Situation an deren Oberfläche gezeigt. Dabei wird der Effekt nicht-linearer Randbedingungen auf die Dynamik von Temperaturfeldern durch das Strahlungsgesetz beachtet. Die gefundenen Gesetzmäßigkeiten und spezifischen Eigenschaften der Ausbildung von Temperaturfeldern bilden die Grundlage zur Identifikation des Wärmeübergangskoeffizienten und der Temperaturabhängigkeit der Temperaturleitfähigkeit.

МЕТОД ПЕРЕМЕЩЕНИЯ ИЗОТЕРМ В ТЕОРИИ И ПРАКТИКЕ ИССЛЕДОВАНИЯ ТЕПЛОМАССОПЕРЕНОСА—I. КИНЕМАТИКА ТЕМПЕРАТУРНЫХ ПОЛЕЙ

Аннотация—Процесс нестационарной теплопроводности в телах без фазового и с фазовым переходом вещества рассмотрен в терминах перемещения изотермических поверхностей. Выведены формулы для определения скорости их перемещения в полупространстве и в ограниченных телах при постоянных и при зависящих от температуры теплофизических характеристиках. При этом установлен новый общий признак регуляризации тепловой кинетики, заключающийся в независимости скорости перемещения изотерм от времени. Выявлена также независимость этой величины в центральных частях ограниченных тел и от тепловой обстановки на их наружной поверхности. Рассмотрено влияние нелинейных граничных условий по закону излучения на кинематику температурных полей. Установленные закономерности и особенности формирования температурных полей положены в основу идентификации коэффициента теплоотдачи и зависимости коэффициента температуропроводности от температуры.